

DESY 00-004

January 2000

ELEMENTS OF BARYOGENESIS*

W. Buchmüller and S. Fredenhagen

Deutsches Elektronen-Synchrotron DESY, 22603 Hamburg, Germany

Abstract

Basic ingredients of the theory of baryogenesis are reviewed with emphasis on out-of-equilibrium decays of heavy particles. The present use of kinetic theory is explained and some attempts to go beyond the classical Boltzmann equations are discussed.

*Presented at the International School of Astrophysics D. Chalonge, Erice, December 1999

1 Scenarios for baryogenesis

One of the main successes of the standard early-universe cosmology is the prediction of the abundances of the light elements, D, ^3He , ^4He and ^7Li . Agreement between theory and observation is obtained for a certain range of the parameter η , the ratio of baryon density and photon density [1],

$$\eta = \frac{n_B}{n_\gamma} = (1.5 - 6.3) \times 10^{-10}, \quad (1)$$

where the present number density of photons is $n_\gamma \sim 400/\text{cm}^3$. Since no significant amount of antimatter is observed in the universe, the baryon density yields directly the cosmological baryon asymmetry, $n_B \simeq n_B - n_{\bar{B}}$.

A matter-antimatter asymmetry can be dynamically generated in an expanding universe if the particle interactions and the cosmological evolution satisfy Sakharov's conditions [2], i.e.

- baryon number violation
- C and CP violation
- deviation from thermal equilibrium .

Although the baryon asymmetry is just a single number, it provides an important relationship between the standard model of cosmology, i.e. the expanding universe with Robertson-Walker metric, and the standard model of particle physics as well as its extensions.

At present there are a number of viable scenarios for baryogenesis. They can be classified according to the different ways in which Sakharov's conditions are realized. Already in the standard model C and CP are not conserved. Also baryon number (B) and lepton number (L) are violated by instanton processes [3]. In grand unified theories B and L are broken by the interactions of gauge bosons and leptoquarks. This is the basis of the classical GUT baryogenesis [4]. Analogously, the L violating decays of heavy Majorana neutrinos lead to leptogenesis [5]. In supersymmetric theories the existence of approximately flat directions in the scalar potential leads to new possibilities. Coherent oscillations of scalar fields may then generate large asymmetries [6].

The crucial departure from thermal equilibrium can also be realized in several ways. One possibility is a sufficiently strong first-order electroweak phase transition [7]. In this case CP violating interactions of the standard model or its supersymmetric extension could in principle generate the observed baryon asymmetry. However, due to the rather large lower bound on the Higgs boson mass of about 105 GeV, which is imposed by the LEP experiments, this interesting possibility is now restricted to a very small range of parameters in the supersymmetric standard model. In the case of the Affleck-Dine scenario the baryon asymmetry is generated at the end of an

inflationary period as a coherent effect of scalar fields which leads to an asymmetry between quarks and antiquarks after reheating [8]. For the classical GUT baryogenesis and for leptogenesis the departure from thermal equilibrium is due to the deviation of the number density of the decaying heavy particles from the equilibrium number density. How strong this deviation from thermal equilibrium is depends on the lifetime of the decaying heavy particles and the cosmological evolution. Further scenarios for baryogenesis are described in [9].

The theory of baryogenesis involves non-perturbative aspects of quantum field theory and also non-equilibrium statistical field theory, in particular the theory of phase transitions and kinetic theory. A crucial ingredient is the connection between

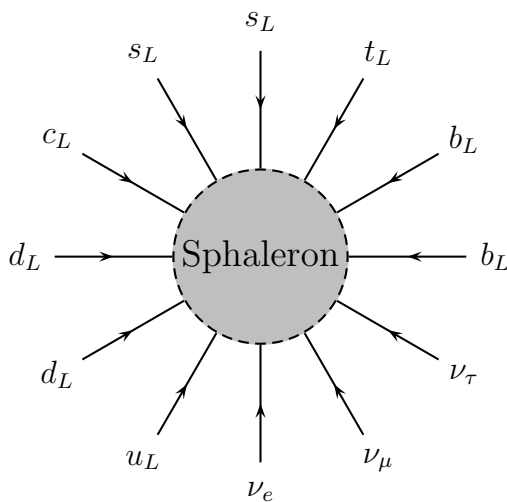


Figure 1: *One of the 12-fermion processes which are in thermal equilibrium in the high-temperature phase of the standard model.*

baryon number and lepton number in the high-temperature, symmetric phase of the standard model. Due to the chiral nature of the weak interactions B and L are not conserved. At zero temperature this has no observable effect due to the smallness of the weak coupling. However, as the temperature approaches the critical temperature T_{EW} of the electroweak transition, B and L violating processes come into thermal equilibrium [10].

The rate of these processes is related to the free energy of sphaleron-type field configurations which carry topological charge. In the standard model they lead to an effective interaction of all left-handed fermions [3] (cf. fig. 1),

$$O_{B+L} = \prod_i (q_{Li} q_{Li} q_{Li} l_{Li}) , \quad (2)$$

which violates baryon and lepton number by three units,

$$\Delta B = \Delta L = 3 . \quad (3)$$

The evaluation of the sphaleron rate in the symmetric high temperature phase is a challenging problem [11]. Although a complete theoretical understanding has not yet been achieved, it is generally believed that B and L violating processes are in thermal equilibrium for temperatures in the range

$$T_{EW} \sim 100 \text{ GeV} < T < T_{SPH} \sim 10^{12} \text{ GeV} . \quad (4)$$

Sphaleron processes have a profound effect on the generation of the cosmological baryon asymmetry, in particular in connection with lepton number violating interactions between lepton and Higgs fields,

$$\mathcal{L}_{\Delta L=2} = \frac{1}{2} f_{ij} l_{Li}^T \phi C l_{Lj} \phi + \text{h.c.} . \quad (5)$$

Such an interaction arises in particular from the exchange of heavy Majorana neutrinos (cf. fig. 2). In the Higgs phase of the standard model, where the Higgs field acquires a vacuum expectation value, it gives rise to Majorana masses of the light neutrinos ν_e , ν_μ and ν_τ .

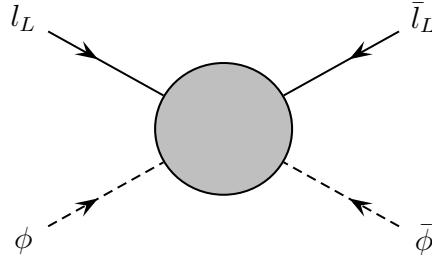


Figure 2: *Effective lepton number violating interaction.*

Eq. (3) suggests that any $B+L$ asymmetry generated before the electroweak phase transition, i.e., at temperatures $T > T_{EW}$, will be washed out. However, since only left-handed fields couple to sphalerons, a non-zero value of $B+L$ can persist in the high-temperature, symmetric phase if there exists a non-vanishing $B-L$ asymmetry. An analysis of the chemical potentials of all particle species in the high-temperature phase yields a relation between the baryon asymmetry $Y_B = (n_B - n_{\bar{B}})/s$, where s is the entropy density, and the corresponding $B-L$ asymmetry Y_{B-L} , respectively [12],

$$Y_B = C Y_{B-L} = \frac{C}{C-1} Y_L . \quad (6)$$

The number C depends on the other processes which are in thermal equilibrium [13]. If these are all standard model interactions one has $C = (8N+4)/(22N+13)$ for N generations. If instead of the Yukawa interactions of the right-handed electron the $\Delta L = 2$ interactions (5) are in equilibrium one finds $C = -2N/(2N+3)$.

The interplay between the sphaleron processes (fig. 1) and the lepton number changing processes (fig. 2) leads to an intriguing relation between neutrino properties and the cosmological baryon asymmetry. The decay of heavy Majorana neutrinos can quantitatively account for the observed asymmetry.

2 Heavy particle decays in a thermal bath

Let us now consider the simplest possibility for a departure from thermal equilibrium, the decay of heavy, weakly interacting particles in a thermal bath. To be specific, we choose the heavy particle to be a Majorana neutrino $N = N^c$ which can decay into a lepton Higgs pair $l\phi$ and also into the CP conjugate state $\bar{l}\bar{\phi}$

$$N \rightarrow l \phi, \quad N \rightarrow \bar{l} \bar{\phi}. \quad (7)$$

In the case of CP violating couplings a lepton asymmetry can be generated in the decays of the heavy neutrinos N which is then partially transformed into a baryon asymmetry [5] by sphaleron processes [10]. Compared to other scenarios of baryogenesis this leptogenesis mechanism has the advantage that, at least in principle, the resulting baryon asymmetry is entirely determined by neutrino properties.

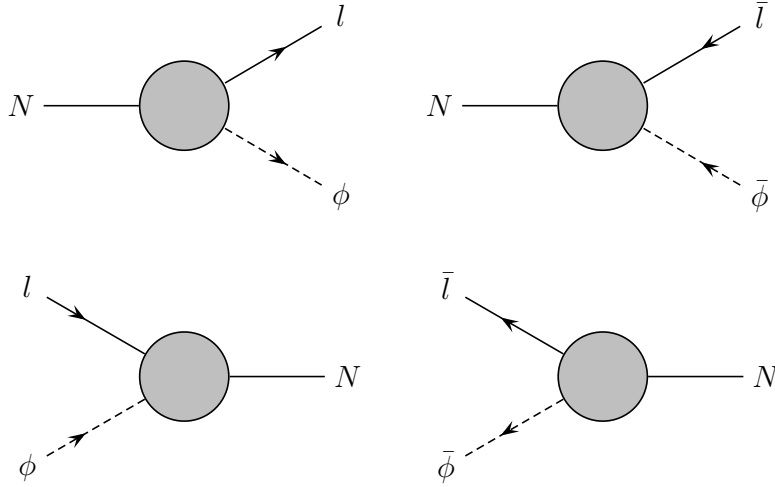


Figure 3: $\Delta L = 1$ processes: decays and inverse decays of a heavy Majorana neutrino.

The generation of a baryon asymmetry is an out-of-equilibrium process which is generally treated by means of Boltzmann equations. A detailed discussion of the basic ideas and some of the subtleties has been given in [14]. The main processes in the thermal bath are the decays and the inverse decays of the heavy neutrinos (cf. fig. 3), and the lepton number conserving ($\Delta L = 0$) and violating ($\Delta L = 2$) processes (cf. fig. 4).

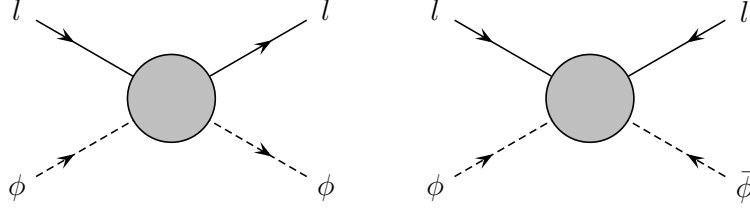


Figure 4: $\Delta L = 0$ and $\Delta L = 2$ lepton Higgs processes.

In addition there are other processes, in particular those involving the t-quark, which are also important in a quantitative analysis [15,16]. A lepton asymmetry can be dynamically generated in an expanding universe if the partial decay widths of the heavy neutrino do not respect CP symmetry,

$$\Gamma(N \rightarrow l\phi) = \frac{1}{2}(1 + \epsilon)\Gamma, \quad \Gamma(N \rightarrow \bar{l}\bar{\phi}) = \frac{1}{2}(1 - \epsilon)\Gamma, \quad (8)$$

where Γ is the total decay width and the parameter $\epsilon \ll 1$ measures the amount of CP violation.

The Boltzmann equations for the number densities of heavy neutrinos (n_N), leptons (n_l) and antileptons ($n_{\bar{l}}$) corresponding to the processes in figs. 3 and 4 are given by

$$\begin{aligned} \frac{dn_N}{dt} + 3Hn_N &= -\gamma(N \rightarrow l\phi) + \gamma(l\phi \rightarrow N) \\ &\quad -\gamma(N \rightarrow \bar{l}\bar{\phi}) + \gamma(\bar{l}\bar{\phi} \rightarrow N), \end{aligned} \quad (9)$$

$$\begin{aligned} \frac{dn_l}{dt} + 3Hn_l &= \gamma(N \rightarrow l\phi) - \gamma(l\phi \rightarrow N) \\ &\quad + \gamma(\bar{l}\bar{\phi} \rightarrow l\phi) - \gamma(l\phi \rightarrow \bar{l}\bar{\phi}), \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{dn_{\bar{l}}}{dt} + 3Hn_{\bar{l}} &= \gamma(N \rightarrow \bar{l}\bar{\phi}) - \gamma(\bar{l}\bar{\phi} \rightarrow N) \\ &\quad + \gamma(l\phi \rightarrow \bar{l}\bar{\phi}) - \gamma(\bar{l}\bar{\phi} \rightarrow l\phi), \end{aligned} \quad (11)$$

with the reaction rates

$$\gamma(N \rightarrow l\phi) = \int d\Phi_{123} f_N(p_1) |\mathcal{M}(N \rightarrow l\phi)|^2, \dots \quad (12)$$

$$\gamma(l\phi \rightarrow \bar{l}\bar{\phi}) = \int d\Phi_{1234} f_l(p_1) f_\phi(p_2) |\mathcal{M}'(l\phi \rightarrow \bar{l}\bar{\phi})|^2, \dots \quad (13)$$

Here H is the Hubble parameter, $d\Phi_{1\dots n}$ denotes the phase space integration over particles in initial and final states,

$$d\Phi_{1\dots n} = \frac{d^3p_1}{(2\pi)^3 2E_1} \dots \frac{d^3p_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^4(p_1 + \dots - p_n), \quad (14)$$

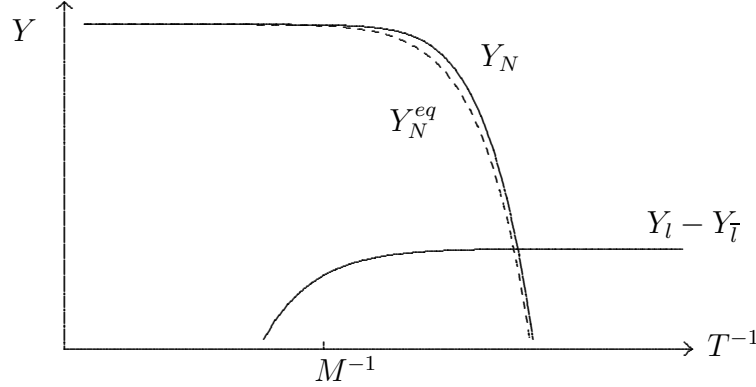


Figure 5: *Time evolution of the number density to entropy density ratio. At $T \sim M$ the system gets out of equilibrium and an asymmetry is produced.*

and

$$f_i(p) = \exp(-\beta E_i(p)) , \quad n_i(p) = g_i \int \frac{d^3p}{(2\pi)^3} f_i(p) , \quad (15)$$

are Boltzmann distribution and number density of particle $i = N, l, \phi$ at temperature $T = 1/\beta$, respectively. \mathcal{M} and \mathcal{M}' denote the scattering matrix elements of the indicated processes at zero temperature; the prime indicates that for the $2 \rightarrow 2$ processes the contribution of the intermediate resonance state has been subtracted. For simplicity we have used in eqs. (12) and (13) Boltzmann distributions rather than Bose-Einstein and Fermi-Dirac distributions, and we have also neglected the distribution functions in the final state which is a good approximation for small number densities. Subtracting (11) from (10) yields the Boltzmann equation for the asymmetry $n_l - n_{\bar{l}}$.

A typical solution of the Boltzmann equations (9) - (11) is shown in fig. 5. Here the ratios of number densities and entropy density,

$$Y_X = \frac{n_X}{s} , \quad (16)$$

are plotted, which remain constant in an expanding universe in thermal equilibrium. A heavy neutrino, which is weakly coupled to the thermal bath, falls out of thermal equilibrium at temperatures $T \sim M$ since its decay is too slow to follow the rapidly decreasing equilibrium distribution $f_N \sim \exp(-\beta M)$. This leads to an excess of the number density, $n_N > n_N^{eq}$. CP violating partial decay widths then yield a lepton asymmetry which, by means of sphaleron processes, is partially transformed into a baryon asymmetry.

The Boltzmann equations are classical equations for the time evolution of number densities. The collision terms, however, are S -matrix elements which involve quantum mechanical interferences of different amplitudes in a crucial manner. Since these

scattering matrix elements are evaluated at zero temperature, one may worry to what extent the quantum mechanical interferences are affected by interactions with the thermal bath. Another subtlety is the separation of the $2 \rightarrow 2$ matrix elements into a resonance contribution and remainder [14],

$$|\mathcal{M}(l\phi \rightarrow \bar{l}\bar{\phi})|^2 = |\mathcal{M}'(l\phi \rightarrow \bar{l}\bar{\phi})|^2 + |\mathcal{M}_{res}(l\phi \rightarrow \bar{l}\bar{\phi})|^2, \quad (17)$$

where the resonance contribution has the form

$$\mathcal{M}_{res}(l\phi \rightarrow \bar{l}\bar{\phi}) \propto \mathcal{M}(l\phi \rightarrow N)\mathcal{M}(N \rightarrow \bar{l}\bar{\phi})^* = |\mathcal{M}(l\phi \rightarrow N)|^2. \quad (18)$$

The entire effect of baryon number generation crucially depends on this separation. The particles which participate in the $2 \rightarrow 2$ processes are massless, hence their distribution functions always coincide with the equilibrium distribution. Only the resonances, treated as on-shell particles, fall out of thermal equilibrium and can then generate an asymmetry in their decays. General theoretical arguments require cancellations between these two types of contributions which we illustrate in the following with two examples.

Cancellations in thermal equilibrium

If all processes, including those which violate baryon number, are in thermal equilibrium the baryon asymmetry vanishes. This is a direct consequence of the *CPT* invariance of the theory,

$$\begin{aligned} \langle B \rangle &= \text{Tr}(\rho B) = \text{Tr}\left((CPT)(CPT)^{-1} \exp(-\beta H) B\right) \\ &= \text{Tr}\left(\exp(-\beta H)(CPT)^{-1} B(CPT)\right) = -\text{Tr}(\rho B) = 0. \end{aligned} \quad (19)$$

Hence, no asymmetry can be generated in equilibrium, and the transition rate which determines the change of the asymmetry has to vanish,

$$\frac{d(n_l - n_{\bar{l}})}{dt} + 3H(n_l - n_{\bar{l}}) = \Delta\gamma^{eq} = 0, \quad (20)$$

where the superscript *eq* denotes rates evaluated with equilibrium distributions.

From eqs. (8), (10) and (11) one obtains for the resonance contribution, i.e. decay and inverse decay,

$$\Delta\gamma_{res}^{eq} = -2\epsilon\gamma^{eq}(N \rightarrow l\phi). \quad (21)$$

This means in particular that the asymmetry generated in the decay is not compensated by the effect of inverse decays. On the contrary, both processes contribute the same amount.

The rate $\Delta\gamma_{res}^{eq}$ has to be compensated by the contribution from $2 \rightarrow 2$ processes which is given by

$$\Delta\gamma_{2\rightarrow 2}^{eq} = 2 \int d\Phi_{1234} f_l^{eq}(p_1) f_{\phi}^{eq}(p_2) \left(|\mathcal{M}'(l\phi \rightarrow \bar{l}\bar{\phi})|^2 - |\mathcal{M}'(\bar{l}\bar{\phi} \rightarrow l\phi)|^2 \right). \quad (22)$$

For weakly coupled heavy neutrinos, i.e. $\Gamma \propto \lambda^2 M$ with $\lambda^2 \ll 1$, this compensation can be easily shown using the unitarity of the S -matrix.

The sum over states in the unitarity relation,

$$\sum_X \left(|\mathcal{M}(l\phi \rightarrow X)|^2 - |\mathcal{M}(X \rightarrow l\phi)|^2 \right) = 0, \quad (23)$$

can be restricted to two-particle states to leading order in the case of weak coupling λ . This implies for the considered $2 \rightarrow 2$ processes,

$$\sum'_{l,\phi,\bar{l}',\bar{\phi}'} \left(|\mathcal{M}(l\phi \rightarrow \bar{l}'\bar{\phi}')|^2 - |\mathcal{M}(\bar{l}'\bar{\phi}' \rightarrow l\phi)|^2 \right) = 0, \quad (24)$$

where the summation \sum' includes momentum integrations under the constraint of fixed total momentum. From eqs. (17) and (24) one obtains

$$\Delta\gamma_{2 \rightarrow 2}^{eq} = 2 \int d\Phi_{1234} f_l^{eq}(p_1) f_{\phi}^{eq}(p_2) \left(-|\mathcal{M}_{res}(l\phi \rightarrow \bar{l}\bar{\phi})|^2 + |\mathcal{M}_{res}(\bar{l}\bar{\phi} \rightarrow l\phi)|^2 \right). \quad (25)$$

In the narrow width approximation, i.e. to leading order in λ^2 , this yields the wanted result,

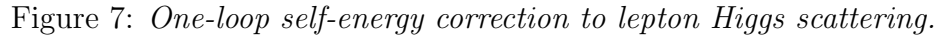
$$\begin{aligned} \Delta\gamma_{2 \rightarrow 2}^{eq} &= 2 \int d\Phi_{1234} f_l^{eq}(p_1) f_{\phi}^{eq}(p_2) \left(-|\mathcal{M}(l\phi \rightarrow N)|^2 |\mathcal{M}(N \rightarrow \bar{l}\bar{\phi})|^2 \right. \\ &\quad \left. + |\mathcal{M}(\bar{l}\bar{\phi} \rightarrow N)|^2 |\mathcal{M}(N \rightarrow l\phi)|^2 \right) \frac{\pi}{M\Gamma} \delta(s - M^2) \\ &= 2\epsilon\gamma^{eq}(N \rightarrow l\phi) = -\Delta\gamma_{res}^{eq}. \end{aligned} \quad (26)$$

This cancellation also illustrates that the Boltzmann equations treat resonances as on-shell real particles. Off-shell effects require a different formalism which will be discussed in Sec. 3.

Cancellations at zero temperature

The lepton asymmetry which is obtained by solving the Boltzmann equations depends crucially on the separation of the $2 \rightarrow 2$ scattering amplitudes into resonance contribution and remainder. How to perform this separation appears obvious for a graph like fig. 6 which represents the interference between the tree level and a one-loop correction term for the vertex $\lambda N l \phi$, together with a s -channel N -propagator.

The identification of the resonance contribution shown in fig. 7 is less obvious. One expects mixing effects in the case of several heavy neutrinos which have been discussed in [17,18,19]. Treating the one-loop self-energy like the one-loop vertex correction yields indeed a finite contribution to the CP asymmetry [18,19] which is of the same order as the vertex contribution. However, one may worry about the fact



It turns out that the resummed propagator does not contribute to the CP asymmetry of $2 \rightarrow 2$ processes at fixed external momenta [21] (fig. 8). Furthermore, even

Figure 8: *Propagator contribution to CP asymmetry for fixed external momenta.*

10

$$\left| \begin{array}{c} l \\ \phi \end{array} \right|^2 - \left| \begin{array}{c} \bar{l} \\ \bar{\phi} \end{array} \right|^2 = 0 .$$

Figure 9: *Total CP asymmetry to leading non-trivial order in the coupling.*

The cancellation of the various contributions to the CP asymmetry follows directly from unitarity. In fact, this is the physical meaning of eq. (24),

$$\sum'_{l,\phi,\bar{l},\bar{\phi}} \left(|\mathcal{M}(l\phi \rightarrow \bar{l}\bar{\phi})|^2 - |\mathcal{M}(\bar{l}\bar{\phi} \rightarrow l\phi)|^2 \right) = 0 .$$

Away from resonance poles, where ordinary perturbation theory holds, the CP asymmetry vanishes to order λ^6 . Corrections due to four-particle intermediate states are $\mathcal{O}(\lambda^8)$. In the resonance region the CP asymmetry vanishes to order λ^2 with corrections $\mathcal{O}(\lambda^4)$.

These cancellations between various contributions to the CP asymmetry demonstrate the importance of identifying correctly the resonance contribution. This is complicated by the fact that different chiral projections of the resummed propagator are diagonalized by different unitary matrices [21]. These matrices, together with the vertex corrections, determine the effective $Nl\phi$ vertex (fig. 10), where N now corresponds to a pole of the full heavy neutrino propagator.

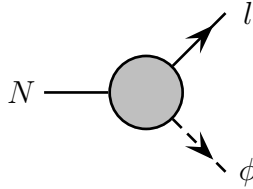


Figure 10: *Effective $Nl\phi$ vertex including mixing effects.*

Given the effective $Nl\phi$ vertex it is straightforward to determine the CP asymmetry in the decay of a heavy Majorana neutrino,

$$\epsilon_i = \frac{\Gamma(N_i \rightarrow l\phi) - \Gamma(N_i \rightarrow \bar{l}\bar{\phi})}{\Gamma(N_i \rightarrow l\phi) + \Gamma(N_i \rightarrow \bar{l}\bar{\phi})} . \quad (27)$$

To leading order in λ^2 , the asymmetry is a sum of two terms, a mixing contribution ($K = \lambda^\dagger \lambda$),

$$\epsilon_i^M = -\frac{1}{8\pi} \sum_{j \neq i} \frac{M_i M_j}{M_i^2 - M_j^2} \frac{\text{Im}\{K_{ij}^2\}}{K_{ii}} , \quad (28)$$

which is directly related to the one-loop self energy [19,18], and the familiar vertex contribution

$$\epsilon_i^V = -\frac{1}{8\pi} \sum_j \frac{\text{Im}\{K_{ij}^2\}}{K_{ii}} f\left(\frac{M_j^2}{M_i^2}\right), \quad (29)$$

where

$$f(x) = \sqrt{x} \left(1 - (1+x) \ln\left(\frac{1+x}{x}\right) \right). \quad (30)$$

These results hold for sufficiently large mass splittings, i.e. $|M_i - M_j| \gg |\Gamma_i - \Gamma_j|$. For small mass differences one expects an enhancement of the mixing contribution [23]. At present, however, the influence of the thermal bath on this enhancement is unclear.

The use of classical Boltzmann equations with collision terms given by S -matrix elements which crucially involve quantum interferences is unsatisfactory. Like the collision terms also the time evolution of the system should be treated quantum mechanically.

3 Quantum mechanics of baryogenesis

One would like to have a full quantum mechanical treatment of baryogenesis. Starting with a density matrix with no initial asymmetry the emergence of an asymmetry should be seen from the full time evolution. The Boltzmann equations should then follow in some limit as a first-order approximation.

The Boltzmann equations are an on-shell approximation: between the interaction processes the particles propagate on-shell, and for the scattering processes the on-shell S -matrix elements are used. A full quantum treatment will also take off-shell effects into account.

Of course, it is hard to obtain a full quantum description of the processes as the whole system is out of equilibrium. We shall therefore discuss a toy model for a relaxation process, following Joichi, Matsumoto and Yoshimura [24], and compare the exact description with the Boltzmann approach.

The model consists of a single quantum mechanical oscillator coupled to a thermal bath of oscillators. The evolution of the system is governed by the Hamiltonian

$$H = Ec^\dagger c + \int_{\omega_c}^{\infty} d\omega \int d\Omega \, \omega b^\dagger(\Omega, \omega) b(\Omega, \omega) \quad (31)$$

$$+ \int_{\omega_c}^{\infty} d\omega \int d\Omega \, \left(\zeta(\Omega, \omega) b^\dagger(\Omega, \omega) c + \text{h.c.} \right). \quad (32)$$

E is the frequency of the single oscillator, the frequencies of the bath oscillators ω are bounded below by ω_c with $E > \omega_c$. Ω is an additional discrete or continuous label for the bath oscillators, and $\zeta(\Omega, \omega)$ denotes the complex coupling.

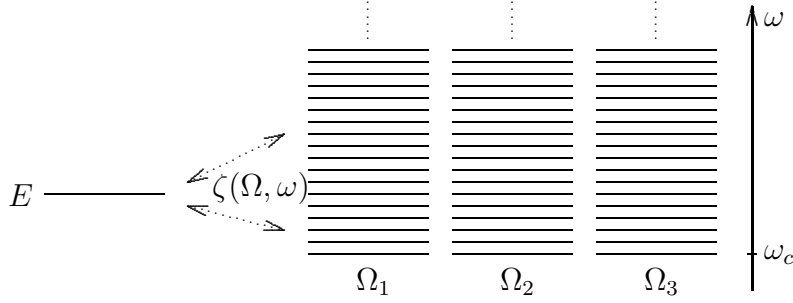


Figure 11: *One single oscillator with frequency E is coupled to a large thermal bath of oscillators.*

The operators c and b fulfil canonical commutation relations,

$$[c, c^\dagger] = 1 \quad , \quad [c, b^\dagger(\Omega, \omega)] = 0 \quad , \quad (33)$$

$$[b(\Omega, \omega), b^\dagger(\Omega', \omega')] = \delta(\Omega - \Omega')\delta(\omega - \omega') \quad . \quad (34)$$

The Hamiltonian is quadratic in c and $b(\Omega, \omega)$ and hence solvable by a change of variables. It is possible to give an explicit formula for new operators $B(\Omega, \omega)$ which diagonalize the Hamiltonian,

$$H = \int_{\omega_c}^{\infty} d\omega \int d\Omega \, \omega B^\dagger(\Omega, \omega) B(\Omega, \omega) \quad . \quad (35)$$

These operators B are obtained as a linear combination of c and b . For ω away from E it takes the form

$$B(\Omega, \omega) = b(\Omega, \omega) + \mathcal{O}(\zeta c, \zeta^* \zeta b) \quad . \quad (36)$$

The time evolution in the new variables is just the free one,

$$B^\dagger(\Omega, \omega, t) = e^{i\omega t} B^\dagger(\Omega, \omega) \quad . \quad (37)$$

By inverting the change of variables one obtains explicit formulas for the time evolution of c and b . Hence we are able to discuss exactly the properties of the system. Let us start with a simple example.

Decay process

Assume that the system is prepared in the initial state $|\psi\rangle = c^\dagger|0\rangle$ where only the single oscillator is excited. We expect that this excited state decays. This should be seen from the time evolution of the occupation number of the single oscillator which is

$$\langle\psi|c^\dagger(t)c(t)|\psi\rangle = |g(t)|^2 \quad , \quad (38)$$

where

$$g(t) = \int d\omega \sigma(\omega) \frac{1}{(-\omega + E - \Pi(\omega))^2 + (\pi\sigma(\omega))^2} e^{i\omega t} . \quad (39)$$

Here $\sigma(\omega)$ is the absolute value squared of the coupling summed over the internal label Ω ,

$$\sigma(\omega) = \int d\Omega |\zeta(\Omega, \omega)|^2 . \quad (40)$$

We see that g is essentially the Fourier transform of the c -propagator including the "self energy" $\Pi(\omega) + i\pi\sigma(\omega)$. For weak coupling, i.e. $\Pi(E) \ll E$ and $\sigma(E) \ll E - \omega_c$, the integrand in the expression for g has a sharp Breit-Wigner resonance at $\omega = E$. The contribution from this resonance is

$$g_0(t) \propto e^{-\Gamma t/2 + iEt} , \quad (41)$$

where $\Gamma = 2\pi\sigma(E)$. This is the result we would expect from the Boltzmann equations: exponential decay with decay rate Γ . But this is not the only contribution. There is a second contribution from the threshold which at large times only decreases with a power law,

$$g_1(t) \propto \kappa \frac{\Gamma(\alpha + 1)}{(E - \omega_c)^2} \frac{1}{t^{\alpha+1}} e^{i(\omega_c t + \alpha\pi/2)} . \quad (42)$$

The constants κ, α parameterize the threshold behaviour for ω close to ω_c ,

$$\sigma(\omega) = \kappa(\omega - \omega_c)^\alpha . \quad (43)$$

At large times this contribution dominates the resonance contribution. So we are led to the interesting result that the exponential Boltzmann decay is only relevant at intermediate times, whereas the asymptotic behaviour is described by a power law. This behaviour holds generally in field theory [25].

Thermal equilibrium

We will now study another interesting case, the situation of thermal equilibrium. The density matrix is then given by

$$\rho = e^{-\beta H} , \quad \beta = \frac{1}{T} , \quad (44)$$

which leads to the usual Bose-Einstein distribution in the variables B ,

$$\langle B^\dagger(\Omega, \omega) B(\Omega', \omega') \rangle = \delta(\Omega - \Omega') \delta(\omega - \omega') \frac{1}{e^{\beta\omega} - 1} . \quad (45)$$

By expressing c in terms of the operators B we obtain the expectation value of the occupation number of the single oscillator,

$$\langle c^\dagger c \rangle = \int_{\omega_c} d\omega \sigma(\omega) \frac{1}{(-\omega + E - \Pi(\omega))^2 + (\pi\sigma(\omega))^2} \frac{1}{e^{\beta\omega} - 1} . \quad (46)$$

For weak coupling we have again a sharp resonance near $\omega = E$. The contribution from this resonance to the occupation number is

$$\langle c^\dagger c \rangle \approx \frac{1}{e^{\beta E} - 1} . \quad (47)$$

This is the contribution we would expect for a free oscillator. However, at small temperatures the resonance contribution is exponentially suppressed and again the threshold behaviour becomes more important, yielding

$$\langle c^\dagger c \rangle \approx \begin{cases} \kappa \frac{\zeta(\alpha+1)\Gamma(\alpha+1)}{(E-\omega_c)^2} T^{\alpha+1} & , \quad \text{for } \omega_c \ll T \ll E , \\ \kappa \frac{\Gamma(\alpha+1)}{(E-\omega_c)^2} e^{-\beta\omega_c} T^{\alpha+1} & , \quad \text{for } T \ll \omega_c < E . \end{cases}$$

Again we obtain a surprising result: at small temperatures the suppression of the occupation number is much weaker than what is expected from the Bose-Einstein distribution.

What are the implications of this result? In [26] it has been argued that the power behaviour significantly affects the WIMP abundance. However, the quantitative importance of this effect requires further investigations [27,28].

What are the implications for baryogenesis and leptogenesis? How large are the errors for the results obtained with standard Boltzmann equations? To answer this question we try to apply our simple model to the case of heavy particle decay. For simplicity we consider only scalar particles. Let X be a heavy particle that may decay into light scalar particles a and b . The idea is to identify the heavy particle X with the single oscillator and the decay products a and b with the thermal oscillator bath. The interaction is given by a Yukawa like coupling between the three scalar fields, $H_{\text{int}} = \lambda \int d^3x \phi_X \phi_a \phi_b$.

This interaction part should have the same structure as the interaction part of the oscillator model. To see how this identification works we expand the fields in Fourier modes. As in the oscillator model we enforce that total particle number is conserved, i.e. we only take into account those parts of the Hamiltonian which describe decay and inverse decay,

$$\begin{aligned} H_{\text{int}} &= \lambda \int d^3x \phi_X \phi_a \phi_b \\ &\rightarrow \lambda \int \frac{d^3q}{(2\pi)^3 (2\omega_q)^{1/2}} \frac{d^3k_a}{(2\pi)^3 (2\omega_a)^{1/2}} \frac{d^3k_b}{(2\pi)^3 (2\omega_b)^{1/2}} (2\pi)^3 \delta^{(3)}(\vec{q} - \vec{k}_a - \vec{k}_b) \\ &\quad \left(c^\dagger(\vec{q}) b_a(\vec{k}_a) b_b(\vec{k}_b) + \text{h.c.} \right) \\ &\stackrel{!}{=} \int \frac{d^3q}{(2\pi)^3} \int d\omega d\Omega \zeta(\Omega, \omega, \vec{q}) \left(c^\dagger(\vec{q}) b(\Omega, \omega, \vec{q}) + \text{h.c.} \right) . \end{aligned} \quad (48)$$

One then reads off the definition of composite operators,

$$b^\dagger(\Omega, \omega, \vec{q}) = \left(\frac{1}{(2\pi)^3} \left| \frac{\partial(\vec{k}_a, \vec{k}_b)}{\partial(\Omega, \omega, \vec{q})} \right| \right)^{1/2} b_a^\dagger(\vec{k}_a) b_b^\dagger(\vec{k}_b) , \quad (49)$$

which are needed for the identification. Here Ω is a label that together with the energy ω and the total momentum \vec{q} determines the momenta \vec{k}_a, \vec{k}_b , e.g. it can be chosen as two angles describing the direction of \vec{k}_a .

These composite operators fulfil commutation relations that differ from the canonical ones. To apply the oscillator model we try to approximate the commutation relation and the kinetic term of the thermal bath as

$$[b(\Omega, \omega, \vec{q}), b^\dagger(\Omega', \omega', \vec{q}')] = (2\pi)^3 \delta(\vec{q} - \vec{q}') \delta(\omega - \omega') \delta(\Omega - \Omega') , \quad (50)$$

$$H_{\text{bath}} = \int \frac{d^3 q}{(2\pi)^3} d\omega d\Omega \omega b^\dagger(\Omega, \omega, \vec{q}) b(\Omega, \omega, \vec{q}) . \quad (51)$$

With these adjustments one can compute the decay of the heavy particle X . The approximations made above do not affect the behaviour of X substantially and correspond to a low density approximation. Again we obtain the result of an exponential decay followed by a power law behaviour at large times.

When describing properties of the thermal bath like the time evolution of an asymmetry things become more involved and it is not possible to describe the long-term behaviour in the model. But nevertheless it appears possible to compute the time derivative of the asymmetry correctly.

An important question is how to implement CP violation in the oscillator model without breaking CPT invariance. This can only be done in an extended version of the oscillator model which involves an extra interaction term. In connection with leptogenesis this corresponds to effects of the heavy neutrinos N_2 and N_3 .

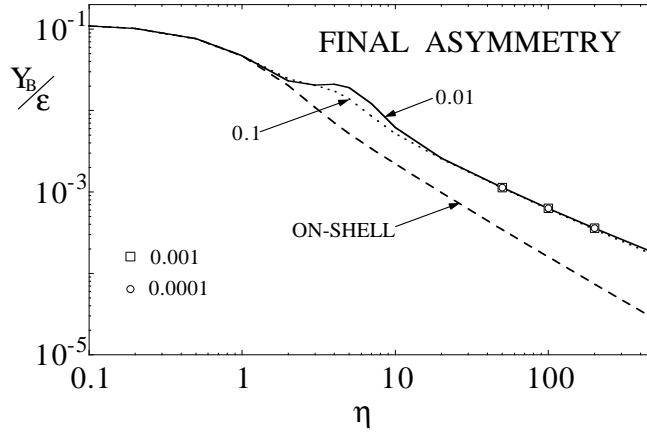


Figure 12: The final asymmetry as function of $\eta = \frac{\Gamma}{H} \Big|_{T=M}$ for two values of $\frac{\Gamma}{M}$: 0.0001 (sharp resonance) and 0.1 (broad resonance).

The production rate of the asymmetry can now be calculated. The resonance contribution coincides exactly with the rate given in section 2 including both contri-

butions (28) and (29) to the CP asymmetry. A detailed analysis of these results is given in [29].

Despite the various remaining problems it is worthwhile to estimate the size of the corrections for the baryogenesis scenario. In [24] such an estimate is given under certain assumptions and approximations. The result is shown in fig. 12. If the decay rate Γ is of order the Hubble rate H or smaller there is no significant deviation from the result obtained with the Boltzmann equations (on-shell result). For $\Gamma/H > 1$, where the asymmetry is smaller because the system is closer to equilibrium, the asymmetry production is enhanced by off-shell effects.

4 Conclusions

At present there still exists a variety of mechanisms which, at least in principle, can account for the cosmological baryon asymmetry. Particularly successful is the leptogenesis scenario. Given the experimental indications for neutrino masses it naturally explains the observed order of magnitude of the cosmological baryon asymmetry without any fine tuning of parameters.

One is therefore led to examine the theoretical basis for current estimates of the baryon asymmetry. Although our understanding of the high-temperature symmetric phase of the standard model has significantly improved in recent years the quantitative description of an out-of-equilibrium process remains a difficult problem.

Particularly subtle is the use of the classical Boltzmann equations together with collision terms derived from S -matrix elements which involve quantum interferences in a crucial manner. A full quantum mechanical treatment which includes the time evolution of the system is highly desirable. At present several interesting ideas are pursued by different groups but a fully satisfactory solution of the problem still remains to be found.

References

- [1] Review of Particle Physics, Eur. Phys. J. **C3** (1998) 1
- [2] A. D. Sakharov, JETP Lett. **5** (1967) 24
- [3] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 8; Phys. Rev. **D 14** (1976) 3422
- [4] M. Yoshimura, Phys. Rev. Lett. **41** (1978) 281; *ibid.* **42** (1979) 746 (E);
S. Dimopoulos, L. Susskind, Phys. Rev. **D 18** (1978) 4500;
D. Toussaint, S. B. Treiman, F. Wilczek, A. Zee, Phys. Rev. **D 19** (1979) 1036;
S. Weinberg, Phys. Rev. Lett. **42** (1979) 850
- [5] M. Fukugita, T. Yanagida, Phys. Lett. **B 174** (1986) 45
- [6] I. Affleck, M. Dine, Nucl. Phys. **B 249** (1985) 361
- [7] For reviews and references, see
V. A. Rubakov, M. E. Shaposhnikov, Phys. Usp. **39** (1996) 461;
Z. Fodor, *Electroweak Phase Transitions*, hep-lat/9909162
- [8] M. Dine, L. Randall, S. Thomas, Nucl. Phys. **B 458** (1996) 291;
B. A. Campbell, M. K. Gaillard, H. Murayama, K. A. Olive, Nucl. Phys. **B 538**
(1999) 351
- [9] For reviews and references, see
A. D. Dolgov, Phys. Rep. **222C** (1992) 309;
A. Riotto, M. Trodden, *Recent Progress in Baryogenesis*, hep-ph/9901362
- [10] V. A. Kuzmin, V. A. Rubakov, M. E. Shaposhnikov, Phys. Lett. **B 155** (1985) 36
- [11] P. Arnold, D. Son, L. Yaffe, Phys. Rev. **D 55** (1997) 6264;
D. Bödeker, Phys. Lett. **B 426** (1998) 351;
J. Ambjorn, A. Krasnitz, Nucl. Phys. **B 506** (1997) 387
- [12] J. A. Harvey, M. S. Turner, Phys. Rev. **D 42** (1990) 3344
- [13] For a recent review and references, see
W. Buchmüller, M. Plümacher, *Neutrino Masses and the Baryon Asymmetry*,
DESY 99-187
- [14] E. W. Kolb, S. Wolfram, Nucl. Phys. **B 172** (1980) 224; Nucl. Phys. **B 195**
(1982) 542(E)
- [15] M. A. Luty, Phys. Rev. **D 45** (1992) 455

- [16] M. Plümacher, Z. Phys. **C 74** (1997) 549; Nucl. Phys. **B 530** (1998) 207
- [17] J. Liu, G. Segrè, Phys. Rev. **D 48** (1993) 4609; Phys. Rev. **D 49** (1994) 1342
- [18] M. Flanz, E. A. Paschos and U. Sarkar, Phys. Lett. **B 345** (1995) 248;
Phys. Lett. **B 384** (1996) 487 (E); M. Flanz, E. A. Paschos, Phys. Rev. **D 58** (1998) 11309
- [19] L. Covi, E. Roulet, F. Vissani, Phys. Lett. **B 384** (1996) 169
- [20] M. Veltman, Physica **29** (1963) 186
- [21] W. Buchmüller, M. Plümacher, Phys. Lett. **B 431** (1998) 354
- [22] E. Roulet, L. Covi, F. Vissani, Phys. Lett. **B 424** (1998) 101
- [23] M. Flanz, E. A. Paschos, U. Sarkar, J. Weiss, Phys. Lett. **B 389** (1996) 693;
L. Covi, E. Roulet, Phys. Lett. **B 399** (1997) 113;
A. Pilaftsis, Nucl. Phys. **B 504** (1997) 61; Phys. Rev. **D 56** (1997) 5431;
R. Rangarajan, H. Mishra, hep-ph/9908417
- [24] I. Joichi, S. Matsumoto, M. Yoshimura, Phys. Rev. **D 58** (1998) 43507
- [25] S. M. Alamoudi, D. Boyanovsky, H. J. de Vega, R. Holman, Phys. Rev. **D 59** (1998) 025003
- [26] S. Matsumoto, M. Yoshimura, Phys. Rev. **D 59** (1999) 123511
- [27] A. Singh, M. Srednicki, Phys. Rev. **D 61** (2000) 023509
- [28] M. Yoshimura, private communication
- [29] S. Fredenhagen, *CP-Verletzung und Asymmetrieentwicklung in einem exakt lösbaeren Modell*, DESY-THESIS-1999-023